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String Kaluza-Klein cosmologies with RR-fields*

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Abstract

We construct 4-dimensional cosmological FRW–models by compactifying a black 5-brane solution of type IIB supergravity, which carries both magnetic NS-NS-charge and RR-charge. The influence of nontrivial RR-fields on the dynamics of the cosmological models is investigated.

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1 Introduction

Cosmological models obtained by dimensional reduction of 10-dimensional supergravity theories are an active field of research [1]–[12]. Since supergravity is the low energy limit of string theory, these models should provide a way towards understanding the quantum effects in cosmology. So far, there has been much activity in investigating models including fields of the NS-NS-sector. The discovery, that the conformal field theory of p-brane-like superstring backgrounds charged under RR-gauge fields is that of open strings with Dirichlet boundary conditions [13, 14], has made the RR-field contents of type II string theories a very interesting subject for string cosmological models.

Recently, in a first study of cosmological models including RR-fields, it was found that inflating phases can occur in a certain range of parameters which label the solutions to the equations of motion [15]. The NS-NS-3-form field strength had been set to zero in the model and the compactification ansatz was chosen to contain a number of maximally symmetric flat subspaces. The two branches of the solutions were separated and curvature singularities appeared in each branch.

In this paper we construct curved 4-dimensional cosmological models containing a NS-NS-3-form field strength as well as nontrivial type IIB RR-fields. To construct the 4-dimensional model, we use a 5-brane-solution of type IIB supergravity whose RR-sector consists of a 3-form and a scalar. By SL(2,R)-invariance of the action, which is realized as a SL(2,Z)-subgroup in IIB superstring theory, we generate 5-brane-solutions with nontrivial RR-fields and compactify the transverse coordinates on a 5-torus, giving a black hole in 5 dimensions. After generalizing the black hole solution to constant curvature solutions we compactify to 4 dimensions, obtaining isotropic FRW-metrics together with nontrivial RR-fields.

The outline of the paper is as follows: In section 2 we introduce the SL(2,R)-transformed 5-brane-solution of type IIB supergravity and compactify it on a 5-torus. In section 3 we make one further compactification to 4 dimensions and obtain FRW-models including nontrivial RR-fields in this way. In section 4 we investigate the influence of the RR-fields on the cosmological models with a detailed analysis of the inflationary phase. Section 5 is devoted to the extremal limit of the model.

2 The 5-brane-solution

Our starting point is the bosonic part of the 10-dimensional SL(2,R)-invariant type IIB supergravity action [16]

$$S_{10} = \int d^{10}x\sqrt{G}\left\{e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - \frac{1}{12}H_1^2\right) - \frac{1}{2}(\partial\chi)^2 - \frac{1}{12}(H_1\chi + H_2)^2\right\}. \qquad (1)$$

In this action the string frame metric G_{MN}, the dilaton Φ and the three form field strength $H_1 = dB_1$ are the familiar fields contained in the NS-NS sector of all 10-dimensional string theories. The three form field strength $H_2 = dB_2$ and the scalar χ belong to the RR-sector. The bosonic field content of IIB superstring theory also includes a selfdual five form field strength, which we have set to zero in order to be able to write a covariant action. A 5-brane-solution to this action is [17, 18]

$$ds^2_{(10)} = -\frac{1-\left(\frac{r_+}{r}\right)^2}{1-\left(\frac{r_-}{r}\right)^2}dt^2 + \frac{dr^2}{\left(1-\left(\frac{r_+}{r}\right)^2\right)\left(1-\left(\frac{r_-}{r}\right)^2\right)} + r^2 d\Omega_3^2 + \delta_{ij}dx^i dx^j \qquad (2)$$

for the 10-dimensional line element in the string frame and

$$\Phi = -\frac{1}{2}\ln\left(1-(r_-/r)^2\right), \quad H_1 = Q\epsilon_3, \quad H_2 = \chi = 0, \qquad (3)$$

for the remaining background fields. The magnetic charge is $Q = 2r_+r_-$ and $d\Omega_3$ and ϵ_3 are the line element and volume element of a 3-sphere, respectively, i.e

$$\begin{aligned} d\Omega_3 &= d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\varphi^2) \\ \epsilon_3 &= \sin^2\psi\sin\theta d\psi\wedge d\theta\wedge d\varphi. \end{aligned} \qquad (4)$$

The RR-fields H_2 and χ are not excited in this solution. We use SL(2,R)-invariance of the action (1) to generate nontrivial RR-fields, making the solution (2,3) an intrinsic IIB solution.

2.1 The SL(2,R)-transformed 5-brane

A manifest SL(2,R)-invariant form of (1) was given in [16, 19] and also used there to construct SL(2,R)-transformed 5-brane-solutions (see also [20, 21, 22] for black hole solutions of type II stringtheory). We do not Hodge-dualize the RR-3-form here, instead we construct a solution possessing two magnetic charges.

Following [16], the action of the SL(2,R)-matrix $\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $ad - bc = 1$ on the background fields is

$$\begin{aligned} \chi' &= \frac{bd + ace^{-2\Phi}}{d^2 + c^2 e^{-2\Phi}} \\ e^{-\Phi'} &= \frac{e^{-\Phi}}{d^2 + c^2 e^{-2\Phi}} \\ H_1' &= dH_1 \\ H_2' &= -bH_1. \end{aligned} \tag{5}$$

This transformation clearly includes S-duality for the choice $a = d = 0, -b = c = 1$, which interchanges the two 3-forms and inverts the string coupling $g_s = e^{\Phi} \to 1/g_s$. In detail, the transformed scalar fields are

$$\begin{aligned} \chi' &= \frac{bd + ac(1 - r_-^2/r^2)}{d^2 + c^2(1 - r_-^2/r^2)} \\ \Phi' &= -\frac{1}{2}\ln\left(1 - (r_-/r)^2\right) + \ln\left(d^2 + c^2\left(1 - (r_-/r)^2\right)\right) \end{aligned} \tag{6}$$

and the string frame line element changes to

$$ds'^2_{(10)} = \sqrt{d^2 + c^2(1 - r_-^2/r^2)}\, ds^2_{(10)}. \tag{7}$$

The Einstein frame metric is not changed by the SL(2,R) transformation.

2.2 The 5-dimensional black hole

We compactify this solution to five dimensions by wrapping the five transverse coordinates of the 5-brane over a 5-torus, i.e. we make the ansatz for the string frame metric

$$ds^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu + e^A \delta_{ij} dx^i dx^j, \tag{8}$$

where $\hat{g}_{\mu\nu}$ is a 5-dimensional Lorentz metric, x^i are coordinates on the 5-torus and A depends on the x^μ only. We neglect the moduli coming from compactifying the three forms as well as vector fields coming from the metric. The effective 5-dimensional action is

$$\begin{aligned} S_5 &= \int d^5x \sqrt{\hat{g}} \left\{ e^{-\phi}\left(\hat{R} + (\partial\phi)^2 - \frac{1}{5}(\partial\sigma)^2 - \frac{1}{12}H_1^2\right) - \right. \\ &\quad \left. - \frac{1}{2}e^{\sigma}\left((\partial\chi)^2 + \frac{1}{6}(H_1\chi + H_2)^2\right) \right\}, \end{aligned} \tag{9}$$

where we have used the rescaled modulus field $\sigma = \frac{5}{2}A$ and the 5-dimensional dilaton ϕ is given by $\phi = 2\Phi - \sigma$. From the transformed 5-brane-solution (6, 7) we get a 5-dimensional solution

$$
\begin{aligned}
ds_5^2 &= \sqrt{d^2 + c^2\left(1 - (r_-/r)^2\right)}\Bigg[- \frac{1 - \left(\frac{r_+}{r}\right)^2}{1 - \left(\frac{r_-}{r}\right)^2} dt^2 + \frac{dr^2}{\left(1 - \left(\frac{r_+}{r}\right)^2\right)\left(1 - \left(\frac{r_-}{r}\right)^2\right)} + \\
&\qquad + r^2 d\Omega_3^2 \Bigg] \\
H_1 &= dQ\epsilon_3, \qquad H_2 = -bQ\epsilon_3 \\
\phi &= \ln \frac{\left(d^2 + c^2\left(1 - (r_-/r)^2\right)\right)^{3/4}}{1 - (r_-/r)^2} \\
\sigma &= \frac{5}{2} \ln \left(d^2 + c^2\left(1 - (r_-/r)^2\right)\right) \\
\chi &= \frac{bd + ac\left(1 - (r_-/r)^2\right)}{d^2 + c^2\left(1 - (r_-/r)^2\right)}.
\end{aligned}
\tag{10}
$$

For Λ the identity transformation, i.e. vanishing RR-fields H_2 and χ, this is just the 5-dimensional black hole solution of [18], written in the string frame. In these equations r_+ and r_- are the values of the parameter r at the outer and inner horizon, respectively. The metric has a curvature singularity at $r = 0$ and also at the spacelike surface $r = r_-$ whereas r_+ is a regular event horizon. This is a black hole solution for $r_+ \geq r_-$ only, in the other case the solution describes a naked singularity. A particular interesting solution is the extremal limit at $r_+ = r_-$. We will discuss the non-extremal and extremal cases in a cosmological context in the next sections.

3 4-dimensional FRW-cosmologies

Following [23], where this solution was generalized to constant curvature spaces, we can produce a 4-dimensional cosmological solution from this black hole by Kaluza-Klein compactification of the time coordinate and re-interpretation of the radius coordinate as 4-dimensional time. In order to get the right signature after compactification we must change signature from $(-,+,+,+,+)$ to $(+,-,+,+,+)$ in the 5-dimensional solution. The generalization of (10) to constant curvature spaces is given by

$$
ds_5^2 = \sqrt{d^2 + c^2\left(1 - (t_-/t)^2\right)}\Bigg[\frac{-k + \left(\frac{t_+}{t}\right)^2}{1 - \left(\frac{t_-}{t}\right)^2} dy^2 - \frac{dt^2}{\left(-k + \left(\frac{t_+}{t}\right)^2\right)\left(1 - \left(\frac{t_-}{t}\right)^2\right)} +
$$

$$
\begin{aligned}
& \quad + t^2 d\Omega_k^2 \Big] \\
H_1 &= dQ\epsilon_k, \qquad H_2 = -bQ\epsilon_k, \qquad Q = 2t_+t_- \\
\phi &= \ln \frac{\left(d^2 + c^2\left(1 - (t_-/t)^2\right)\right)^{3/4}}{1 - (t_-/t)^2} \\
\sigma &= \frac{5}{2} \ln \left(d^2 + c^2\left(1 - (t_-/t)^2\right)\right) \\
\chi &= \frac{bd + ac\left(1 - (t_-/t)^2\right)}{d^2 + c^2\left(1 - (t_-/t)^2\right)},
\end{aligned} \tag{11}
$$

where $k \in \{-1, 0, 1\}$ labels the constant curvature three-spaces with line element and volume element

$$
\begin{aligned}
d\Omega_k &= d\psi^2 + \left(\frac{\sin\sqrt{k}\psi}{\sqrt{k}}\right)^2 (d\theta^2 + \sin^2\theta d\varphi^2), \\
\epsilon_k &= \left(\frac{\sin\sqrt{k}\psi}{\sqrt{k}}\right)^2 \sin\theta d\psi \wedge d\theta \wedge d\varphi.
\end{aligned} \tag{12}
$$

It should be emphasized that the time coordinate in this solution is the radius coordinate of the 5-dimensional black hole and the signature is different from usual black hole physics.

We reduce to 4 dimensions using the ansatz

$$
ds_5^2 = Y^2 dy^2 + ds_4^2 \tag{13}
$$

where the S^1-modulus Y^2 turns out to be

$$
Y^2(t) = \frac{-k + (t_+/t)^2}{1 - (t_-/t)^2} \sqrt{d^2 + c^2\left(1 - t_-^2/t^2\right)}. \tag{14}
$$

By introducing $\rho = \ln Y$ we arrive at the 4-dimensional action

$$
\begin{aligned}
S_4 &= \int d^4x\sqrt{g}\left\{ e^{-2\varphi}\left(R + 4(\partial\varphi)^2 - (\partial\rho)^2 - \frac{1}{5}(d\sigma)^2 - \frac{1}{12}H_1^2\right) - \right. \\
& \quad \left. - \frac{1}{2}e^{\rho+\sigma}\left((\partial\chi)^2 + \frac{1}{6}(H_1\chi + H_2)^2\right)\right\}.
\end{aligned} \tag{15}
$$

The 4-dimensional dilaton φ is given by $\varphi = \frac{1}{2}(\phi - \rho)$. The 5-dimensional black hole (11) reduces to the 4-dimensional FRW-solution

$$
\begin{aligned}
ds_4^2 &= \sqrt{d^2 + c^2\left(1 - t_-^2/t^2\right)} \left(- \frac{dt^2}{\left(-k + \left(\frac{t_+}{t}\right)^2\right)\left(1 - \left(\frac{t_-}{t}\right)^2\right)} + t^2 d\Omega_k^2 \right) \\
H_1 &= dQ\epsilon_k, \quad H_2 = -bQ\epsilon_k, \quad Q = 2t_+t_-
\end{aligned}
$$

$$
\begin{aligned}
\rho &= \frac{1}{2}\ln\left(\frac{\left(-k+\left(\frac{t_+}{t}\right)^2\right)\sqrt{d^2+c^2\left(1-t_-^2/t^2\right)}}{1-\left(\frac{t_-}{t}\right)^2}\right) \\
\varphi &= \frac{1}{4}\ln\left(\frac{d^2+c^2\left(1-t_-^2/t^2\right)}{\left(-k+\left(\frac{t_+}{t}\right)^2\right)\left(1-\left(\frac{t_-}{t}\right)^2\right)}\right) \\
\sigma &= \frac{5}{4}\ln\left(d^2+c^2\left(1-t_-^2/t^2\right)\right) \\
\chi &= \frac{bd+ac\left(1-(t_-/t)^2\right)}{d^2+c^2\left(1-t_-^2/t^2\right)}
\end{aligned} \tag{16}
$$

For the special choice of SL(2,R)-parameters $a=d=1, b=c=0$, the RR-fields H_2 and χ and the T^5-modulus field σ are zero and we recover the cosmological models described in [23]. These models have been studied in detail there, both in the string and Einstein frame. Since the SL(2,R)-transformation does not affect the Einstein metric, we concentrate on the string frame in the following. Let us briefly recall the properties of the model in the pure NS-NS-case, i.e. without contributions from the RR-fields. The solution describes a closed ($k=+1$), open ($k=-1$), or flat ($k=0$) cosmology respectively. In the string frame, for $k=+1$, the three space oscillates[4] between the extrema $t_\pm$. For $k=0,-1$ the solution is not oscillating, the extension of the corresponding three spaces is bounded from below at $t=t_-$ and tends to infinity as $t\to\pm\infty$. The dilaton φ is always divergent at $t=t_-$, corresponding to the minimal extension of the three spaces. For $k=+1$, the dilaton is also divergent at $t=t_+$, while for $k=0,-1$, where the time region has no upper bound, it tends to zero for $t\to\pm\infty$. The S^1-modulus ρ is divergent at $t=t_-$, indicating decompactification to 5 dimensions at this points. For $k=+1$, the S^1-radius becomes zero at the maximum extension of three-space. In the other cases $k=0,-1$, the modulus tends to a constant value as $t\to\pm\infty$.

In the Einstein frame, the curvature scalar is singular at $t=t_-$ in all three cases and also at $t=t_+$ in the case $k=+1$. The extension of three-space shrinks to zero at these points, indicating a big-crunch or big-bang.

4 Effects of the RR-fields on the cosmological models

Let us now study the qualitative changes caused by adding nontrivial RR-fields to the solution described above. As can be seen from the dilaton expression in (16), in the case $d=0$

[4] This can be seen after changing to conformal time by the transformation (25).

one divergence cancels leaving the expression for the dilaton

$$\varphi_{d=0} = \frac{1}{4}\ln\left(\frac{c^2}{-k+\left(\frac{t_+}{t}\right)^2}\right). \tag{17}$$

For $k=+1$, there is still a divergence at $t=t_+$, but the divergence at $t=t_-$ has disappeared in all cases $k=-1,0,+1$. The case $d=0$ corresponds to zero NS-NS-charge and a constant RR-scalar $\chi=-ab$. We want to see how the scale factors of the FRW-models behave in this special case. For this we change to a coordinate system which is usually used to study FRW-cosmologies

$$ds^2 = -d\tau^2 + a^2(\tau)d\Omega_k^2. \tag{18}$$

The comoving time coordinate τ is defined by a solution to the differential equation

$$d\tau^2 = \frac{\sqrt{d^2+c^2\left(1-t_-^2/t^2\right)}}{\left(-k+\left(\frac{t_+}{t}\right)^2\right)\left(1-\left(\frac{t_-}{t}\right)^2\right)}dt^2, \quad t(\tau=0)=t_- \tag{19}$$

and the scale factor $a(\tau)$ is

$$a^2(\tau) = t^2(\tau)\sqrt{d^2+c^2\left(1-t_-^2/t^2(\tau)\right)}. \tag{20}$$

For $d=0$ the scale factor $a(\tau)$ goes to zero at $\tau=0$. Calculating $\dot{a}(\tau)$ we find the two different behaviors

$$\lim_{\tau\to 0}\frac{da(\tau)}{d\tau} = \begin{cases} 0 & : \quad d\neq 0 \\ \infty & : \quad d=0 \end{cases} \tag{21}$$

So by switching off the NS-NS-3-form H_1 completely, we produce a singularity in the string frame scale factor, whereas a finite value of the NS-NS-charge smoothes out this singularity. At the same time we find that the T^5-compactification radius is zero for $d=0$ at $t=t_-$, whereas the S^1-radius is still divergent there. The dilaton φ takes the minimum value at $t=t_-$ for $d=0$ and it diverges there for $d\neq 0$. The magnetic charge of the NS-NS-3-form prevents the universe from collapsing, as was already recognized in [24]. Without NS-NS-charge, the scale factor exhibits a singularity regardless of the existence of RR-charge.

The qualitative behavior of the dilaton-singularities does not change when RR-fields are added to a non-zero NS-NS-field strength. The minimum extension of the universe at $t=t_-$ is $t_-\sqrt{d}$ and for $k=+1$ the maximum extension is given by $t_-\left(d^2+c^2(1-t_-^2/t_+^2\right)^{1/4}$. However, the duration and effectivity of a possible initial inflationary phase depend on the parameters c and d as well as on t_- and t_+, as can be seen from the equations (19) and (20). We will discuss this dependence in detail now.

4.1 The inflationary phase

In the presence of a NS-NS-charge the string frame scale factor of the cosmology is free of singularities, although in the Einstein frame the familiar curvature singularity close to the time $t = t_-$ appears. It is interesting to investigate how this feature is affected by adding RR-fields and whether a phenomenologically interesting inflationary phase can occur. From a phenomenological point of view one can argue that the Einstein frame is the preferred coordinate system. On the other hand one can treat both frames on equal footing, as has been done in recent years in numerous analyses where the transition of a superinflationary pre-big bang phase into a a FRW type cosmology was worked out in the string frame [5]–[11].

In the following we will examine in more detail the effect of RR-fields on the string frame scale factor close to the time $t = t_-$, where the influence of these fields are most apparent. It was pointed out in [23] that in the asymptotic limit $t \to t_-$ the expansion scales like $a(\tau) \sim \tau^2 + const.$ and from the discussion in the previous section we know that the S-dual model contains a singularity in the scale factor at this point. One can easily imagine that close to $\tau = 0$ a short phase of superluminal expansion might occur in the sense of power-law inflation with $a(\tau) \sim \tau^p$ and p sufficiently large to ensure an expansion that could be of phenomenological interest.

As a measure to describe an accelerated expansion of the universe we use the quantity $\zeta \equiv \ddot{a}/a = \dot{H} + H^2 = \frac{p(p-1)}{\tau^2}$. Here H is the Hubble parameter $H = \dot{a}/a$, a dot means the derivative with respect to τ and the last part of the equation holds if the expansion is of power law type, $a(\tau) \sim \tau^p$. In the general case we are not able to find analytical expressions for the quantity ζ, because equation (19) can not be integrated directly. Instead we will give numerical solutions for a certain choice of parameters in fig. 1. It is shown how the acceleration changes with transformations that lie within a SO(2,R)-subgroup of SL(2,R), parametrized by the rotation angle β. An increasing rotation angle corresponds to an enlargement of the fields of the RR-sector and simultaneously a reduction of the NS-NS-3-form field strength. We observe that the initial acceleration is very much enhanced by exchanging NS-NS-charge by RR-fields but a subsequent decelerating phase becomes also gradually more important and thus taking off much of what was gained in the accelerating phase. At the S-dual point we encounter again the transition from a non-singular to the singular universe and $\zeta \to -\infty$. The number of e-foldings $\mathcal{N}$ produced during the expansion increases only slightly by enlarging the RR-fields and remains of the order of one for almost all values of β. Since $\mathcal{N}$ must exceed a certain limit as a prerequisite for a viable inflationary scenario, none of the cosmologies fulfills this requirement.

An expression for the number of e-foldings produced between the time t_- and t can easily

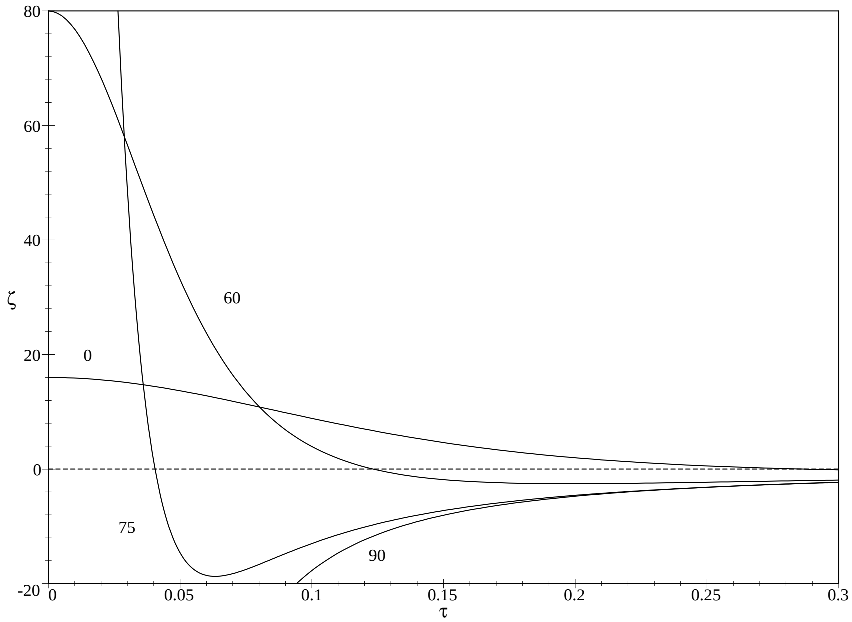


Figure 1: $\zeta \equiv \ddot{a}/a = \dot{H} + H^2$ for several SO(2)-rotations. (Rotation angles in degrees, 0 means zero RR-charge, 90 means zero NS-NS charge.) The spatial section was assumed to be flat ($k = 0$) and the time parameters were chosen to be $t_- = 1$ and $t_+ = 4$.

be derived from (20)

$$\mathcal{N}(t;c) = \ln\left(\frac{t}{t_-}\right) + \frac{1}{4}\ln\left(d + \frac{c^2}{d}\left(1 - \left(\frac{t_-}{t}\right)^2\right)\right). \tag{22}$$

Since the string frame metric is only influenced by the SL(2,R)-parameters c and d, we study the case where for a fixed value of d (fixed NS-NS-charge) the parameter c is increased. For the specific choice $a = d = 1, b = 0$ and c a free parameter, it becomes apparent that the RR-fields enhances the number of e-folds only logarithmically. However, for t_- being very close to zero, $\mathcal{N}$ can become sufficiently large for having cosmological significance. Upon expanding $a(\tau)$ to second order in τ it can also be seen how the singular case is recovered,

$$a(\tau) = t_-\sqrt{d} - \frac{1}{2t_-\sqrt{d}}\left(k - \left(\frac{t_+}{t_-}\right)^2\right)\left(1 + \frac{1}{2}\frac{c^2}{d^2}\right)\tau^2 + \mathcal{O}(\tau^4). \tag{23}$$

The pre-factor of the quadratic term gets large for small t_- and d and for large c. Also, the behavior of ζ with this choice of parameters is qualitatively the same as shown in fig. 1

for the SO(2) transformed models, as well as for all other sets that have been investigated. Moreover, curved spatial sections ($k = \pm 1$) have only little influence on the qualitative picture.

5 Extremal limit

It was shown in [23] that for vanishing RR-fields the 5-dimensional solution (11) has an extremal limit $t_+ = kt_-$, where the theory corresponds to a WZW model[5] and is therefore exact to all orders in α'. Whereas for p-brane superstring backgrounds charged under RR gauge fields the corresponding conformal field theory is known to be that of open strings with Dirichlet boundary conditions [13, 14], the higher order contributions to the tree level equations of motions for non-p-brane like backgrounds still lack an explicit sigma model description [25]. For this the extremal limit described above may not remain exact after performing an SL(2,R) transformation. The corresponding 4d solution, valid for arbitrary k, is

$$\begin{aligned}
ds^2 &= t_-^2 \sqrt{d^2 + c^2 \left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}\, t_-}\right)^2} \left(-d\eta^2 + d\Omega_k^2\right) \\
\rho &= \frac{1}{2} \ln \left(\sqrt{d^2 + c^2 \left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}\, t_-}\right)^2} \left(\frac{\sqrt{k}}{\tan\sqrt{k}\eta}\right)^2 \right) \\
\varphi &= -\frac{1}{2} \ln \left(\frac{\cos\sqrt{k}\eta \sin\sqrt{k}\eta}{\sqrt{k} t_-^2 \sqrt{d^2 + c^2 \left(\frac{\sin\sqrt{k}\eta}{\sqrt{k} t_-}\right)^2}} \right) \\
\sigma &= \frac{5}{4} \ln \left(d^2 + c^2 \left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}\, t_-}\right)^2 \right) \\
\chi &= \frac{bd + ac \left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}\, t_-}\right)^2}{d^2 + c^2 \left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}\, t_-}\right)^2} \\
H_1 &= 2d\sqrt{\pm k}\, t_-^2 \epsilon_k, \quad H_2 = -2b\sqrt{\pm k}\, t_-^2 \epsilon_k
\end{aligned} \tag{24}$$

In this solution, η is the conformal time defined by the transformation

$$t^2 = t_-^2 + (t_+^2 - kt_-^2) \left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}}\right)^2 \tag{25}$$

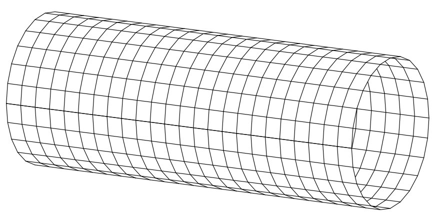
[5] In this limit the theory decouples in a direct product of a 3d (spherical) part and a 2d (η, y) part. For $k = +1, -1$, the 2d part corresponds to a $SL(2,R)/U(1)$ coset model and for $k = 0$ it is trivial.

and a possible constant part of the 5-dimensional dilaton in (11) has to be adjusted [23]

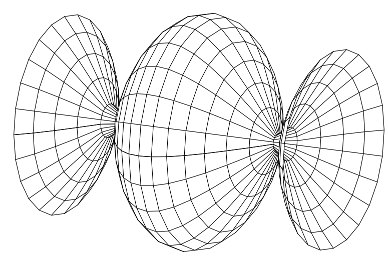
$$\phi_0 \to \phi_0 + \frac{1}{2}\ln(t_+^2 - kt_-^2) \tag{26}$$

in order to be able to perform the extremal limit. For vanishing RR-fields, we recover again the solution given in [23]. In this limit, the geometry is simply $R \times S_k^3$ for all three cases (see figure 2a), where S_k^3 stands for a sphere, pseudo-sphere or flat space respectively. Adding RR-fields to the solution, the universe behaves different for different values of k. For $k = 1$, the conformal factor is oscillating between $|d|$ and $\sqrt{d^2 + c^2/t_-^2}$. In the other cases the universe approaches the minimum size $|d|$ for $\eta = 0$ and then expands for ever. Especially for the S-dual case, where the NS-NS-field H_1 is switched off, the corresponding solutions exhibit again singularities (see figures 2b-d).

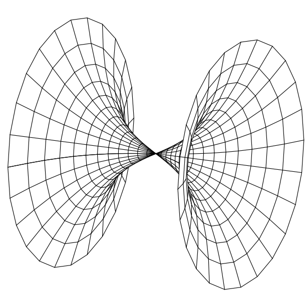
2a



2b



2c



2d

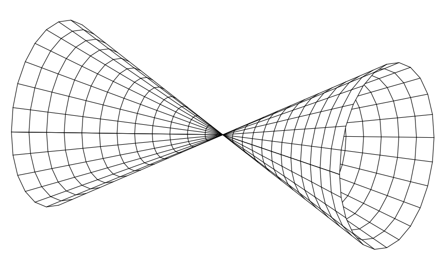


Figure 2: The extremal limit for pure NS-NS-charge (2a) and the extremal limits for pure RR-charge for $k = +1$ (2b), $k = -1$ (2c) and $k = 0$ (2d).

6 Conclusions

We constructed 4-dimensional FRW-cosmologies from 5-dimensional black hole solutions following [23]. Our models contain nontrivial RR-fields, a 3-form and a scalar, which are generated by SL(2,R) transformations of a black 5-brane solution of type IIB supergravity.

The effect of this RR-fields on the resulting cosmologies have been studied. For the S-dual model, where the NS-NS-charge is replaced by the corresponding RR-charge, the dilaton singularity at the minimum extension of spatial 3-space disappears, but the model still decompactifies to 5 dimensions there. Also, other than the NS-NS-field strength, the RR-field strength cannot prevent the universe from collapsing, and we find a singularity in the string frame scale factor, which is absent when the NS-NS-charge is different from zero.

It was further shown that the influence of RR-fields on the cosmic expansion results in a large increase of the initial accelerating phase. It is however followed by a subsequent deceleration phase that increases in a similar way. For that reason the number of e-folds produced during the combined phases increases only slightly, almost logarithmically, with the RR-fields being turned on. Thus, such models do not seem to be suitable for an explanation of spatial homogeneity and isotropy, especially that of the CMB radiation, based on inflationary cosmology. However, for a small fraction of parameter space, the superluminal expansion of the physical space is effective enough to solve some of the problems inflationary cosmology was invented for.

We examined also the extremal limit of the model, for which the 5-dimensional black hole is on the verge of becoming a naked singularity. Since we generalized the 5-dimensional solution to a constant curvature solution, it is not clear whether the successful description of extremal black holes as being composed of non-interacting branes, anti-branes and momentum [26, 27] can be applied to this extremal limit. The effect of RR-fields added to the extremal limit of the pure NS-NS case is to render dynamics to the steady universe. For the S-dual case we find again singularities in the string frame scale factor.

During writing this paper [28] appeared, where also cosmological models from toroidally compactification of type II theories are discussed.

7 Acknowledgment

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